Vector Analysis

IFoS (IFS) Previous Year Questions (PYQ) from 2025 to 2009

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2025

1. For vector fields $\vec{u}$ and $\vec{v}$, prove that $\nabla \times (\vec{u} \times \vec{v}) = \vec{u}(\nabla \cdot \vec{v}) - \vec{v}(\nabla \cdot \vec{u}) + (\vec{v} \cdot \nabla)\vec{u} - (\vec{u} \cdot \nabla)\vec{v}$.

[8 Marks]

2. (i) Find the directional derivative of $F(x, y, z) = xy^2 - 4x^2y + z^2$ at $(1, -1, 2)$ in the direction of $6\hat{i} + 2\hat{j} + 3\hat{k}$. Also find its maximum value.

(ii) Verify Stokes' theorem for $\vec{f} = (x+y)\hat{i} + yz^2\hat{j} + y^2z\hat{k}$, where S is the upper surface of the sphere $x^2+y^2+z^2 = 1$ over $z = 0$ and Γ is its boundary in the xy-plane.

[5+10=15]

3. (i) Determine $\vec{a}$ such that $\dfrac{d\hat{T}}{ds} = \vec{a} \times \hat{T}$, $\dfrac{d\hat{N}}{ds} = \vec{a} \times \hat{N}$, $\dfrac{d\hat{B}}{ds} = \vec{a} \times \hat{B}$ represents Frenet-Serret formulae.

(ii) A curve in space is represented by $\vec{r} = e^t \cos t\, \hat{i} + e^t \sin t\, \hat{j} + e^t\, \hat{k}$. Find the curvature and principal normal of this curve at $t = 0$.

[5+10=15]

4. (i) Find the values of α and β such that the vectors $\vec{f} = (\alpha x + y)\hat{i} + (y - 3z)\hat{j} + (x + \alpha z)\hat{k}$ and $\vec{g} = \dfrac{x\hat{i} + y\hat{j}}{x^2 + y^2} + \hat{k}$ are solenoidal.

(ii) Show that the vector $\vec{f} = (2x - yz)\hat{i} + (2y - zx)\hat{j} + (2z - xy)\hat{k}$ is irrotational and find a scalar function ϕ such that $\vec{f} = \text{grad}\,\phi$.

[3+7=10]

2024

5. If $\vec{F}$ is a solenoidal vector, then show that curl curl curl curl $\vec{F} = \nabla^2\nabla^2\vec{F} = \nabla^4\vec{F}$.

[8 Marks]

6. (i) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then find curl $\left(\dfrac{\vec{r}}{|\vec{r}|}\right)$.

(ii) Find the curvature and torsion of the curve $x = a\cos t$, $y = a\sin t$, $z = bt$.

[6+9=15]

7. (i) If ϕ satisfies $\nabla^2\phi = 0$, then show that $\nabla\phi$ is both solenoidal and irrotational.

(ii) Verify the divergence theorem for the function $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ taken over the parallelepiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

[4+11=15]

8. Verify Green's theorem for $\oint_C \left[(xy + y^2)\, dx + x^2\, dy\right]$, where C is bounded by the curves $y = x$ and $y = x^2$.

[10 Marks]

2023

9. If $\nabla \cdot \vec{E} = 0$, $\nabla \cdot \vec{H} = 0$, $\nabla \times \vec{E} = -\dfrac{\partial \vec{H}}{\partial t}$ and $\nabla \times \vec{H} = \dfrac{\partial \vec{E}}{\partial t}$, then show that $\nabla^2\vec{H} = \dfrac{\partial^2\vec{H}}{\partial t^2}$ and $\nabla^2\vec{E} = \dfrac{\partial^2\vec{E}}{\partial t^2}$.

[8 Marks]

10. (i) Show that the area bounded by a simple closed curve C is given by $\dfrac{1}{2}\oint_C (x\, dy - y\, dx)$. Hence obtain the area of an ellipse.

(ii) Evaluate $\int_\Gamma (e^x\, dx + 2y\, dy - dz)$ by using Stokes' theorem, where Γ is the curve $x^2 + y^2 = a^2$, $z = h$.

[8+7=15]

11. (i) Given that $u = x + y + z$, $v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$. Show that the vectors grad u, grad v and grad w are coplanar.

(ii) For the curve given by $\vec{r} = \left(2t, t^2, \frac{t^3}{3}\right)$, find the curvature and torsion at $t = 1$.

[8+7=15]

12. (i) Given that the vectors $\vec{f}$ and $\vec{g}$ are irrotational. Show that the vector $\vec{f} \times \vec{g}$ is solenoidal.

(ii) Show that the vector $\vec{q} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational and find a scalar function ϕ such that $\vec{q} = \text{grad}\,\phi$.

[3+7=10]

2022

13. Determine constants a, b, c so that the directional derivative of $\phi(x, y, z) = axy^2 + byz + cz^2x^3$ at $(1, 2, -1)$ has a maximum magnitude 88 in a direction parallel to the z-axis.

[8 Marks]

14. Given that C is a curve of the intersection of the cylinder $x^2 + y^2 = 4$ and the plane $x + y + z = 2$, and C is described counterclockwise. Verify Stokes' theorem for the line integral $\int_C (-y^3\,dx + x^3\,dy - z^3\,dz)$.

[15 Marks]

15. Derive vector identity for divergence of cross product of two vector point functions. Given a relation between linear and angular velocity as $\vec{v} = \vec{\omega} \times \vec{r}$. If $\vec{\omega}$ is constant, then show that (i) $\text{curl}\,\vec{v} = 2\vec{\omega}$ and (ii) $\text{div}\,\vec{v} = 0$.

[10 Marks]

16. If a curve in space is represented by $\vec{r} = \vec{r}(t)$, then derive expressions of its torsion and curvature in terms of $\vec{r}$, $\dot{\vec{r}}$ and $\ddot{\vec{r}}$. Find the curvature and torsion of the curve given by $\vec{r} = (at - a\sin t,\ a - a\cos t,\ bt)$.

[15 Marks]

2021

17. If $\vec{F} = \left(y\frac{\partial\phi}{\partial z} - z\frac{\partial\phi}{\partial y}\right)\hat{i} + \left(z\frac{\partial\phi}{\partial x} - x\frac{\partial\phi}{\partial z}\right)\hat{j} + \left(x\frac{\partial\phi}{\partial y} - y\frac{\partial\phi}{\partial x}\right)\hat{k}$, then prove that $\vec{F} - (\vec{r} \times \nabla\phi) = \vec{F} \cdot \vec{r} = \vec{F} \cdot \nabla\phi = 0$.

[8 Marks]

18. Let $\vec{a}$ and $\vec{b}$ be any two vector point functions defined on Euclidean space $\mathbb{R}^3$. Derive the vector identity for $\nabla(\vec{a} \cdot \vec{b})$. Verify that identity for $\text{grad}(\text{grad}\,\phi \cdot \text{grad}\,\psi)$, where $\phi = 3x^2y$ and $\psi = xz^2 - 2y$.

[15 Marks]

19. State Gauss' Divergence Theorem completely. Verify the theorem for a field vector $\vec{f} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by the cylinder $x^2 + y^2 = 9$, $z = 0$, $z = 4$.

[10 Marks]

20. (i) Prove that principal normals at consecutive points on a curve in space do not intersect unless its torsion is zero.

(ii) Prove that the principal normal of a curve in space will be the binormal of another curve if the curvature of the given curve is proportional to $\kappa^2 + \tau^2$.

[7+8=15]